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Published in:
IAC-20 The Cyber Space Edition

Publication date:
2020

IAC–2020–IAC-20,C1,9,9,x59498

An Energy-Based Approach to Satellite Attitude Control in Presence of Disturbances for a CubeSat Mission

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The aim of this paper is to present a novel control strategy of the satellite attitude control problem on an energy-based setting, more specifically on the port-Hamiltonian framework. Controlling the orientation of a satellite becomes challenging in presence of nonlinear external disturbances such as the gravity-gradient torque and the atmospheric drag, which are external torques coming from the iteration of the spacecraft with external entities. We make use of the advantages of representing the system under study via the port-Hamiltonian framework due to its clear control design philosophy. The structure presented on the energy setting shows the interconnection of energy storage and dissipation elements plus the input and output ports pair, i.e., efforts and flows of the mechanical system. Then, the provided approach attains an asymptotic stable orientation where the key control strategy depends on the orientation and rotation velocity measurements, together with an integral action on the system's output. Furthermore, the advantage of our approach relies on an energy consumption optimization of the controller, together with the lack of linearization strategies due to the modeling-based framework. Consequently, the closed-loop system shows robustness in terms of parameters uncertainty due to the nature of the port-Hamiltonian approach. Moreover, a numerical propagation of the spacecraft attitude states is provided where we have considered a satellite placed in an orbit that experiences gravity gradient and atmospheric drag external torques similarly to the orbit and external torques experienced by the International Space Station's orbit. Here, the perturbations are simulated by propagating both the attitude and the orbit of the spacecraft, with atmospheric drag modeled as a coupled orbit and attitude dependent perturbation. The propagation is done modeled to replicate the conditions of the mission GWSat, a 3-unit CubeSat mission lead by the George Washington University, with the Costa Rica Institute of Technology providing the design of the attitude control system. The latter is done to demonstrate effectiveness of our controller for a realistic scenario.
Keywords: port-Hamiltonian systems, attitude dynamics, perturbation model, scenario analysis.

I. Introduction

The port-Hamiltonian framework (PH) is an energy-based approach. The formalism depends on power ports, energy variables, and their interconnection such that the resulting system has passivity-based properties as presented by van der Schaft (2000); Duindam et al. (2009); van der Schaft and Jeltsema (2014). It is via dissipation and energy elements together with power preserving ports that the transfer of energy between the environment and the system is given. When two or more PH system are interconnected, then the PH structure is preserved. Such is the case for a closed-loop PH system, for instance.

The rigid-body attitude control problem with two different approaches are designed via the PH framework by Forni et al. (2015) and Fujimoto et al. (2015). An energy-balancing passivity-based approach Forni et al. (2015) is provided via rotational matrices in order to achieve a set-point control. Its main contribution is to achieve the desired configuration for a rotational matrix without velocity measurements. Nonetheless, the controller proposed by Forni et al. (2015) becomes ineffective when non-neglected disturbances are given in the systems output. Moreover, in Fujimoto et al. (2015), a trajectory tracking control for the attitude control and the orbital dynamics problems is provided for a non-realistic scenario. In this paper, we consider in our simulation results not only external torques affecting the satellite attitude configuration but also we include a scenario with parameters closer to the dynamics and kinematics of a real spacecraft.

In Muñoz-Arias (2019), a novel controller inspired by Forni et al. (2015) and Dirksz and Scherpen (2011) is proposed, by which a desired attitude kinematics and a attitude dynamics configuration of a satellite system is attained. Nevertheless, the spacecraft scenario is limited in the sense that the perturbations are limited to a series of nonlinear sinusoidal functions inspired by Xiao et al. (2016).

In this paper, a realistic scenario where both the orbit and the attitude of the spacecraft are propagated in order to determine the form of the perturbation is used to test the effectiveness of the PH control strategy. Based on the model proposed in Chaves Jimenez (2020), a scenario based on the initial conditions of a 3-unit CubeSat after it is placed in orbit from the International Space Station is consider. In the model, the satellite is affected by the J_2 effect, the atmospheric drag force and torque, and the gravity gradient. The spacecraft is not considered a point mass, and as such, the forces and torques are the source of coupling between orbit and attitude dynamics. It is proven in this paper that under the aforementioned conditions, the spacecraft is able to achieve asymptotic stability using the PH control strategy.

The paper is structured as follows. In Section 2, we present the reference frame definitions, and the orbit and attitude dynamics of the spacecraft. We then include the perturbation model in Section 3 due to fact that we assume a low-Earth-orbit scenario for the satellite. Furthermore, Section 4 introduces the PH formalism where we specifically addressed the PH approach to the satellite dynamics. Consequently, in Section 5, we present our control law on a energy-based setting by which we attain asymptotic stability. It follows in Section 6 the specific scenario which match the approximate conditions of the International Space Station from where many satellites are placed in orbit. We make use of the scenario and present simulation results in Section 7 which includes external torques as disturbances. Finally, concluding remarks and future work is provided in Section 8.

II. Dynamics Model Definition

II.i Reference Frames Definition

Consider a spacecraft orbiting the Earth. Let $\mathcal{I}$ denote an inertial geocentric Cartesian, right-handed coordinate frame (Wertz, 1984, p. 28). Consider spacecraft centered inertial frames with axes parallel to the geocentered inertial frame $\mathcal{I}$. From this point on $\mathcal{I}$ will be the only considered inertial frame without loss of generality.

Let $\mathcal{B}$ denote the spacecraft-centered Cartesian right-handed coordinates frame with origins at the spacecraft center of mass with the z-axis in the direction of the highest moment of inertia, and the x and y-axes parallel to the area vectors of the faces of the spacecraft (Fig. 1).

II.ii Orbit and Attitude Dynamics

In this section, the dynamics model of the absolute dynamics of a spacecraft used in this paper are given, but not derived, for the sake of simplicity. If further information is required about the derivation of these equations, the reader is referred to Wertz (1984, Ch. 16),Alfriend et al. (2009).

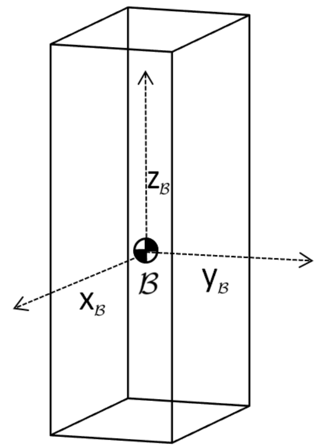

Fig. 1: Definition of the Body Frame ($\mathcal{B}$).

Let $\boldsymbol{\omega}$ denote the angular velocity vector of the frame $\mathcal{B}$ with respect to $\mathcal{I}$ projected on $\mathcal{B}$. The attitude of the spacecraft is denoted using the quaternion parametrization

$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{\varrho} \\ q \end{bmatrix}, \qquad [1]$$

where

$$\boldsymbol{\varrho} = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}^T = \boldsymbol{e}\sin(\theta) \qquad [2]$$

$$q = \cos(\theta) \qquad [3]$$

with $\boldsymbol{e}$ the unit Euler axis and θ the rotation angle and scalar part q Crassidis et al. (2007). Let $\Xi(\boldsymbol{q})$ denote the following 4×3 matrix

$$\Xi(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{e}^{\times} + q\mathbf{1}_3 \\ -\boldsymbol{e}^T \end{bmatrix}, \qquad [4]$$

where $\boldsymbol{e}^{\times}$ denotes the cross-product matrix

$$\boldsymbol{e}^{\times} = \begin{bmatrix} 0 & -e_3 & e_2 \\ e_3 & 0 & -e_1 \\ -e_2 & e_1 & 0 \end{bmatrix}, \qquad [5]$$

and $\mathbf{1}_3$ is the identity matrix in $\mathbb{R}^{3\times 3}$. Let $\boldsymbol{q}$, with vector part $\boldsymbol{e}$ and scalar part q, denote the quaternion of the rotation from $\mathcal{I}$ to $\mathcal{B}_1$. Furthermore, the "vee" map $(\cdot)^{\vee} : so(3)$ denotes the inverse operation of "cross", namely

$$(\boldsymbol{e}^{\times})^{\vee} = \boldsymbol{e}. \qquad [6]$$

For notation purposes, the gradient of a scalar vector is given by

$$\nabla_x := \frac{\delta}{\delta x}. \qquad [7]$$

All vectors are considered as column vectors, and $\mathrm{tr}\,(A)$ is the trace of the matrix $A \in \mathbb{R}^{n\times n}$.

Defining now $\boldsymbol{x}$ as the complete state, under the assumption of rigid body rotations around their centers of mass, the orbital and attitude dynamics is governed by the following system of (Wertz, 1984, Ch. 16),Alfriend et al. (2009), i.e.,

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{r}} \\ \dot{\boldsymbol{v}} \\ \dot{\boldsymbol{q}} \\ \boldsymbol{I}_1\,\dot{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{v} \\ -\frac{\mu}{r^3}\boldsymbol{r} + \boldsymbol{a}_p \\ \frac{1}{2}\,\Xi(\boldsymbol{q})\omega \\ -\omega^{\times}\boldsymbol{I}\omega + \boldsymbol{\tau} \end{bmatrix}, \qquad [8]$$

with $\boldsymbol{I}$ the tensor of inertia of the spacecraft around its center of mass on the $\mathcal{B}$ frame, $\boldsymbol{a}_p$ the accelerations provoked by the perturbation affecting the orbital dynamics and $\boldsymbol{\tau}$ the perturbing torques affecting the attitude dynamics .

III. Perturbation Model

III.i Atmospheric Drag as the Source of Coupling between Attitude and Orbital Dynamics

When a satellite is in low-Earth-orbit, the interaction of the upper atmosphere particles with its surface is the cause of atmospheric drag force and torque. This atmospheric perturbation acts directly opposite to the velocity of the satellite motion with respect to the atmospheric flux, producing a deceleration of the satellite Montenbruck and Gill (2005). This effect constitutes the strongest non-gravitational perturbation of orbital dynamics missions working at an altitude of around 300 km Gill et al. (2013).

Typically, the force model considering atmospheric drag use assumes a constant spacecraft effective area. Nevertheless, in reality, unless the satellite is a perfect sphere, or that spacecraft are controlled so that their effective area are constant, these effective areas change as a function of attitude, meaning that the magnitude of this perturbation on the orbit dynamics is a function of the spacecraft orientation, thus affecting both the orbit and attitude dynamics on spacecraft in flight.

The effect of the atmospheric drag, considered as the main non-gravitational force acting on the spacecraft dynamics, is described in the proper reference frames.

The acceleration due to atmospheric drag may be modeled as

$$\boldsymbol{a}_a = -\frac{1}{2}\frac{C_D\rho(\boldsymbol{r})}{m}A_{ef}\boldsymbol{v}_s^2\hat{\boldsymbol{v}}_s, \qquad [9]$$

where C_D the drag coefficient of the S/C, $\rho(\boldsymbol{r})$ is the atmospheric density, m is the mass of the spacecraft, $\boldsymbol{v}_s$ the velocity of the spacecraft surface with respect to the atmosphere, $\boldsymbol{v}_s = \boldsymbol{v} - \boldsymbol{v}_a$, where $\boldsymbol{v}_a$ is the velocity of the atmosphere.

If a spacecraft is modeled as a number of plane surfaces, the effective area is given by

$$A_{ef} = \sum_{i=1}^{s} A_i(\hat{\boldsymbol{n}}_i^T \cdot \hat{\boldsymbol{v}}_s), \qquad [10]$$

where s is equal to the amount of plane surfaces composing the spacecraft,A_i the magnitude of area i amd $\hat{\boldsymbol{n}}_i$ a unit vector perpendicular to area i. The atmosphere is assumed to be spherical and co-rotating with the Earth Zhong and Gurfil (2013). In this case, its velocity projection in $\mathcal{I}$ is given by

$$\boldsymbol{v}_a = \boldsymbol{\omega_E} \times \boldsymbol{r}, \qquad [11]$$

where $\boldsymbol{\omega_E}$ is the Earth rotation around its own axis. The torque produced by the atmospheric drag is then given by

$$\boldsymbol{\tau}_a = -\frac{1}{2}C_D\rho(\boldsymbol{r})\sum_{i=1}^{k} A_i(\hat{\boldsymbol{n}}_i\boldsymbol{v}_s)(\boldsymbol{d}_i \times \boldsymbol{v}_s), \qquad [12]$$

with $\boldsymbol{d}_i$ the distance between the center of pressure of area i and the center of mass of the S/C.

III.ii J_2 Effect

The J_2 effect projected in $\mathcal{I}$, as described in Junkins and Schaub (2009), is given by

$$\boldsymbol{a}_{J2} = -\frac{3\mu\boldsymbol{r}_{Earth}^2}{2mr^4}J_2\left(\left(1 - 5\frac{r_z^2}{r^2}\right)\frac{\boldsymbol{r}}{r} + 2\frac{r_z}{r}\hat{z}\right) \qquad [13]$$

with m the mass of the spacecraft, $\boldsymbol{r}_{Earth}$ being the Equatorial radius of Earth, r the absolute value of the position $\boldsymbol{r}$, r_z the z-axis component of the $\boldsymbol{r}$ state, $\hat{z} = [0, 0, 1]^T$, μ the gravity coefficient of the Earth, and J_2 the first zonal harmonic for Earth.

III.iii The Gravitational Torque Effect

Any nonsymmetrical object of finite dimensions in orbit is subject to a gravitational torque because of the variation in the Earth's gravitational object over the object Wertz (1984). If a spherical mass distribution of the Earth is assumed, this torque, projected in frame $\mathcal{B}$, is given Wertz (1984) as

$$\boldsymbol{\tau}_g \quad = \quad 3\frac{\mu}{r^5}(\boldsymbol{r}|_{\mathcal{B}})^{\times}(\boldsymbol{I}\ \boldsymbol{r}|_{\mathcal{B}}). \qquad [14]$$

Recall that in this work it has been assumed that $\boldsymbol{r} = \boldsymbol{r}|_{\mathcal{I}}$, which indicates the projection of the position in $\mathcal{I}$. This means that its projection in $\mathcal{B}$, $\boldsymbol{r}|_{\mathcal{B}} = D(\boldsymbol{q})\boldsymbol{r}$ such that

$$\boldsymbol{\tau}_g \quad = \quad 3\frac{\mu}{r^5}(D(\boldsymbol{q})\boldsymbol{r})^{\times}(\boldsymbol{I}D(\boldsymbol{q})\boldsymbol{r}). \qquad [15]$$

Since the perturbation model is now given by [12], [13], and [15], then we introduce in our next section the energy-based setting of the spacecraft dynamics towards the control design strategy.

IV. PORT-HAMILTONIAN FRAMEWORK

In this section, we present the port-Hamiltonian (PH) formalism for a general class of physical systems, and later we present a formulation for the attitude control dynamics. We apply the results of Dirksz et al. (2008) in order to reinforced the proposal proposal of Forni et al. (2015) in front of nonlinear disturbances. The PH framework is based on the description of systems in terms of energy variables, their interconnection structure, and power port pairs.

PH systems include a large family of physical nonlinear systems which includes the dynamics of satellites. The transfer of energy between the physical system and the environment is given through energy elements, dissipation elements and power preserving ports van der Schaft (2000); Duindam et al. (2009); van der Schaft and Jeltsema (2014).

A time-invariant PH system corresponds to the

$$\Sigma \begin{cases} \dot{x} = [\mathcal{J}(x) - \mathcal{R}(x)] \nabla_x H(x) + g(x) u, \\ y = g(x)^\top \nabla_x H(x), \end{cases} \quad [16]$$

where the state variable is given by $x \in \mathcal{R}^{\mathcal{N}}$, and the input-output port-pair representing flows and efforts are given by

$$u \in \mathcal{R}^{\mathcal{N}}, \quad [17]$$

$$y \in \mathcal{R}^{\mathcal{M}}, \quad [18]$$

respectively. Furthermore, the matrices input, interconnection and dissipation matrices of [16] are given by

$$g(x) \in \mathcal{R}^{\mathcal{N}\times\mathcal{M}}, \quad [19]$$

$$\mathcal{J}(x) = -\mathcal{J}(x)^\top, \ \mathcal{J}(x) \in \mathbb{R}^{\mathcal{N}\times\mathcal{N}}, \quad [20]$$

$$\mathcal{R}(x) = \mathcal{R}(x)^\top \geq 0, \ \mathcal{R}(x) \in \mathbb{R}^{\mathcal{N}\times\mathcal{N}}, \quad [21]$$

where $\mathcal{M} \leq \mathcal{N}$ being $\mathcal{M} = \mathcal{N}$ a fully actuated system, and $\mathcal{M} < \mathcal{N}$ an underactuated one. Furthermore, the energy function of system [16] is

$$H(x) \in \mathbb{R}. \quad [22]$$

Differentiating the Hamiltonian along the trajectories of $\dot{x}$, we recover the energy balance

$$\dot{H}(x) = -\nabla_x^\top H(x) \mathcal{R}(x) \nabla_x H(x) + y^\top u \leq y^\top u \quad [23]$$

where we clearly see how we consider the system [16] conservative.

IV.i port-Hamiltonian formulation of satellite (rigid body)

Given a rigid-body in space (satellite), we define its inner energy (Hamiltonian) function as

$$H(q,p) := \frac{1}{2} p^\top \boldsymbol{I}^{-1} p \quad [24]$$

with $x = \mathrm{col}(q,p)$ being the state variable that depends on the (generalized) position $q \in \mathbb{R}^3$, and generalized momenta $p \in \mathbb{R}^3$. Furthermore, the matrix $\boldsymbol{I} := \mathrm{diag}\left(\boldsymbol{I}_x, \boldsymbol{I}_y, \boldsymbol{I}_z\right)$ is the (principal) inertia matrix. Also, $p := \boldsymbol{I}\boldsymbol{\omega}$ being $\boldsymbol{\omega} \in \mathbb{R}^3$ the angular velocity vector. The dynamics of p is then given by

$$\dot{p} = p^\times \nabla_p H(p) + u \quad [25]$$

with $u = \boldsymbol{\tau} \in \mathbb{R}^3$ being the applied control torques to the rigid body (satellite). Based on [24] and [25], we obtain the following PH formulation

$$\Sigma_S \begin{cases} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{9\times3} & r(q) \\ -r(q)^\top & p^\times \end{bmatrix} \begin{bmatrix} \nabla_q H(q,p) \\ \nabla_p H(q,p) \end{bmatrix} + \begin{bmatrix} 0_{9\times3} \\ G(q) \end{bmatrix} u \\ \\ y = G(q)^\top \nabla_p H(q,p) = G(q)^\top \boldsymbol{\omega} \end{cases} \quad [26]$$

where the dissipation matrix is assume zero, i.e. $\mathcal{R}(q,p) = 0$, $G(q) \in \mathbb{R}^{3\times3}$ being the input matrix, and $r(q) : \mathbb{R}^9 \to \mathbb{R}^{9\times3}$ is computed as

$$r(q) := \begin{bmatrix} R_x^\times \\ R_y^\times \\ R_z^\times \end{bmatrix} \quad [27]$$

with

$$q := \mathrm{vec}\left\{R^\top\right\} = \begin{bmatrix} R_x & R_y & R_z \end{bmatrix}. \quad [28]$$

Notice from [26] that the dynamics of the generalized position q is given by

$$\dot{q} = r(q)\, p \quad [29]$$

with $r(q)$ as in [27]. In Forni et al. (2015), it is shown the full derivation of [26] with the matrix $r(q)$ as in [27], and the vector of position coordinates q as in [28].

In the follow up, we present the proposed control law to attain a desired attitude of a system in presence of nonlinear disturbances.

IV.ii Definition of the desired attitude configuration

Inspired by Dirksz et al. (2008) and Donaire and Junco (2009), we make use of an *adapted momenta* strategy where information about the position is added to the momenta coordinate without loosing the system's structure, and at the same time attaining asymptotic stability for control purposes.

First, we define the desired attitude as

$$R_{\text{ref}} := \begin{bmatrix} R_{\text{ref},x} \\ R_{\text{ref},y} \\ R_{\text{ref},z} \end{bmatrix}, \qquad [30]$$

where where $R_{\text{ref},x}$, $R_{\text{ref},y}$, and $R_{\text{ref},z}$, denote the first, second and third, row of R_{ref}, respectively, then we make use of the energy candidate function

$$H_{\text{ref}}(q) := \frac{1}{2}\text{tr}\left[K_p\left(\mathbf{1}_3 - R_{\text{ref}}^\top R(q)\right)\right] \qquad [31]$$

with $K_p = \text{diag}\left(k_{px}, k_{py}, k_{pz}\right) \succ 0$, and $\mathbf{1}_3 \in \mathbb{R}^{3\times 3}$ an identity matrix which Bullo and Lewis (2004) has preliminary suggested. Then, the new Hamiltonian is proposed as

$$H_d(q,p) = H(q,p) + H_{\text{ref}}(q), \qquad [32]$$

and, inspired by Forni et al. (2015), (an auxiliary) matrix is also proposed as

$$R_{\text{aux}}(q) := K_p R_{\text{ref}}^\top R(q) - R(q)^\top R_{\text{ref}} K_p^\top \qquad [33]$$

such a the configuration error to stabilized $\bar{q} \in \mathbb{R}^3$ is given by

$$\bar{q} = r(q)^\top \nabla_q H_{\text{ref}}(q) = \frac{1}{2} R_{aux}(q)^\vee = \begin{bmatrix} -R_{\text{aux},2,3}(q) \\ R_{\text{aux},1,3}(q) \\ -R_{\text{aux},1,2}(q) \end{bmatrix} \qquad [34]$$

with $R_{aux}(q)$ as in [33]. Notice that the new generalized coordinate $\bar{q} \in \mathbb{R}^n$ represents the error given by the reference matrix R_{ref} in [30] or, in other words, the desired attitude configuration.

V. A novel control strategy

Once we have define the error between the current and desired attitude configuration, i.e by [34], then we introduce the adapted momenta as

$$\bar{p} = p + K_p \bar{q} \qquad [35]$$

where $\bar{p} \in \mathbb{R}^n$, with $n > 0$ constant, positive matrix $K_p \succ 0$, and desired configuration error $\bar{q}$ to be defined later on. Then, the resulting output of the new PH systems is

$$\bar{y} = \bar{p}. \qquad [36]$$

that contrary to the original output y of [16], we see how the position becomes also relevant for control purposes.

Here, we also introduce an extended dynamics $z \in \mathbb{R}^n$ with an integral action on the output, i.e.

$$\dot{z} = -K_i \hat{y} \qquad [37]$$

where a tuning matrix $K_i \in \mathbb{R}^{3\times 3}$, such that $K_i \succ 0$.

Based on the new output $\bar{y}$, and the extended dynamics z as in [37], we proposed the following control law.

Theorem 1 *Given a satellite dynamics represented by* [26] *with the generalized coordinate q and the generalized momenta p, we obtain asymptotic stability at the a desired configuration error $\bar{q}$ as in* [34] *with the torque input vector u as*

$$u = -\frac{1}{2}\bar{q} - K_p \bar{y} + z \qquad [38]$$

with a positive constant matrix $K_p \succ 0$, a new system output $\bar{y}$ as in [36]*, and an integral action on the extended dynamics, i.e z as in* [37] *with a positive constant matrix $K_i \succ 0$.*

Proof: Clearly, from [38], the adapted momenta $\bar{p}$ as in [35], the new output $\bar{y}$ as in [36] that depends on the position q and speed $\dot{q}$, together with the integral action on the dynamics of z as in [37], we obtain the closed loop system

$$\Sigma_{CL}\left\{\begin{bmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{I}^{-1} & r(\bar{q}) & 0_{9\times 3} \\ -r(\bar{q})^\top & -K_d & K_i \\ 0_{3\times 9} & -K_i & 0_{3\times 3} \end{bmatrix} \begin{bmatrix} \nabla_{\bar{q}} \bar{H}(\bar{q},\bar{p},z) \\ \nabla_{\bar{p}} \bar{H}(\bar{q},\bar{p},z) \\ \nabla_z \bar{H}(\bar{q},\bar{p},z) \end{bmatrix}\right. \qquad [39]$$

with a Lyapunov Candidate function $\bar{H}(\bar{q},\bar{p},z)$ given by

$$\bar{H}(\bar{q},\bar{p},z) = \frac{1}{2}\bar{p}^\top \bar{p} + \frac{1}{2}\text{tr}\left[K_p\left(\mathbf{1}_3 - R_{\text{ref}}^\top R(\bar{q})\right)\right] + \frac{1}{2} z^\top K_i^{-1} z \qquad [40]$$

First, we see how $\bar{H}(\bar{q},\bar{p},z) \geq 0$, and if we evaluate $\dot{\bar{H}}$ along the trajectories of [39], we obtain that $\dot{\bar{H}}(\bar{q},\bar{p},z) \leq 0$. Finally, since in R_{ref}, $\bar{H}$ has a minimum, then via the Lyapunov Stability theory van der Schaft and Jeltsema (2014), we conclude that the system [39] has a equilibrium point in $(\bar{q},\bar{p},z) = (0,0,0)$. ■

Theorem 1 shows how we can attain asymptotic stability on a desired attitude configuration R_{ref} given by [30]. Even though, it depends on position and velocity measurements, the PH framework ensures robustness in presence of parameters uncertainties and disturbances, as demonstrated in van der Schaft (2000); Donaire and Junco (2009), and van der Schaft and Jeltsema (2014). In the next section, we incorporate such nonlinear disturbances to the system [26] in order to demonstrate the relevance our approach via numerical simulations.

VI. Scenario

Take a spacecraft system orbiting Earth with dynamics described in [8] which follows a circular orbit with an initial altitude of 400 km and an inclination of 51.6°. The altitude and inclinations are selected to match the approximate conditions of the International Space Station (ISS)

Initial Conditions.	
Position (km)	$[400km + R_E, 0, 0]$
Velocity (km/s)	$[0, 7.7142\cos(51.6°), 7.7142\sin(51.6°)]$
Attitude quaternion	$[0, 0, 0, 1]$
Rotation rate (rad/s)	$[0, 2\pi, 0]$
Spacecraft mechanical characteristics.	
Mass (m)	3.6 kg
Inertia matrix ($\boldsymbol{I}$)	$\begin{bmatrix} 0.055 & 0 & 0 \\ 0 & 0.055 & 0 \\ 0 & 0 & 0.017 \end{bmatrix}$ kg m^2
Drag coefficient (C_D)	2.3

Table 1: Configuration of the dynamics of the system under study.

from where many satellites have been placed in orbit. In the chosen orbit a strong perturbation due to the atmospheric drag is obtained. The drag coefficients (C_D) of the spacecraft are assumed to have the same dimensions of the ones used in the simulation of the formation flying mission proposed by TU Delft in the framework of the QB50 mission Gill et al. (2013). The parameters used in the scenario are given on Table 1. It is assumed that the atmospheric density is known which is given by its values at solar radiation maximum (see for instance Larson and Wertz (1992)). Furthermore, a 3-unit *CubeSat* is used on this research. These satellites have a $30 \times 10 \times 10$ cuboid form. It is the spacecraft defined to be use for the GWSat project, project led by the George Washington University, were the Costa Rica Institute of Technology is in charge of the attitude control algorithm design.

Once the modeling approach, control design, and scenario are provided in the aforementioned sections, we present next our simulation results in order to numerically validate our proposed control law.

VII. Simulation results

We first make use of a satellite system modeled in the PH framework as in [26] with external perturbations such as the atmospheric drag as in [12], the J_2 effect as in [13], and the gravity torque as in [15]. Furthermore, given the scenario presented in Section 6 with Table 1, we then apply the control law [38] to the input u of [26]. Our control parameters are $K_i = \frac{1}{10}\mathbf{1}_3$, $K_p = \mathbf{1}_3$, and $K_d = \mathbf{1}_3$ which can always be fine tuned to achieve different performances depending on the desired transient response. Clearly, we can see in Figure 2 how there is a transient response of $t < 25\ s$, and finally the system is stabilized at $t \geq 25\ s$. Consequently, robustness is present in front of nonlinear disturbances which results from the model-based strategy together with the integral action proposed in [37]. An example of the attenuated disturbance during the simulation is shown in Figure 3 which corresponds to atmospheric drag external torque in [12].

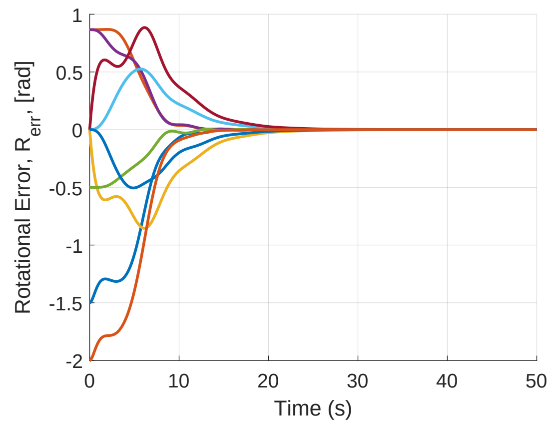

Fig. 2: Simulation results of the attitude configuration for the system [26] **with control law** [38]**, and nonlinear disturbances** [12]**,** [13]**, and** [15]**. Each line represents one of the nine elements of the error matrix defined by** [34]**. Asymptotic stability is obtained after** $t = 25\,s$ **due to the chosen gains for the controller.**

VIII. Concluding remarks and future work

In this paper, it is proven that the energy-based control strategy presented here is effective to achieve asymptotic stability of a satellite in low-Earth orbit, where strong aerodynamic and gravitational perturbations are present, and are modeled as dependent of both the orbit and the attitude dynamics of the spacecraft. The spacecraft is modeled as a set of areas, and as such, the variation of the attitude and orbit of the spacecraft affect the magnitude of the perturbations. Given the advance perturbation model proposed here, this paper might be considered an extension of the paper Muñoz-Arias (2019).

The use of more advanced actuation models will be the subject of future extension of this work. At the same time, energy shaping in for optimization of the use of energy will be considered as a potential extension of the current study. When achieved, these results might have implications in the efficient use of spacecraft attitude actuators such as magnetorquers, rotation wheels and thrusters.

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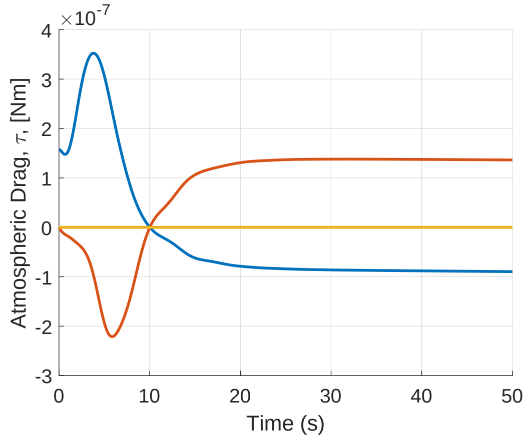

Fig. 3: Perturbation due to the Atmospheric drag torque $\tau_a = \texttt{col}(\tau_{a1}, \tau_{a2}, \tau_{a1})$ given in blue, red, and yellow, respectively. The perturbation vector is projected in $\mathcal{B}$ as in [12]**.**

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